[6]Dirling, R., "A Method for Computing Roughwall Heat Transfer Rates on Reentry Nosetips," AIAA Paper 73-763, Palm Springs, Calif., 1973.

[7]Schlichting, H., *Boundary Layer Theory* (translated by J. Kestin), 4th ed., McGraw-Hill, New York, 1960, p. 320.

[8]Cansdale, J. and McNaughton, I., "Calculation of Surface Temperature and the Accretion Rate in a Mixed Water Droplet/Ice Crystal Cloud," Royal Aircraft Establishment Technical Report 77090, 1977.

Calculation of Transient Thermal Stresses in Solid Rocket Propellants

Thomas L. Cost*
The University of Alabama, University, Alabama

Introduction

RECENTLY, extensive examination of transient thermoviscoelastic theories has occurred.[1,2] Although the thermorheologically simple (TS) theory seems to apply to many solid propellant materials tested at different temperatures when the temperature is constant, experimental measurements of stresses are significantly higher than the stresses predicted by the TS theory when the temperature is changing with time.

To illustrate the inadequacy of the TS theory, consider the test results presented in Fig. 1. Uniaxial specimens of a PBAA propellant were placed in an Instron test machine and simultaneously strained and cooled at a constant rate. The experimental stress was deduced from the measured load and the theoretical predictions were made using TS theory. The TS theory is seen to significantly underpredict the measured stresses.

Hufferd et al.[1] have each proposed modifications to the TS theory to account for the demonstrated deficiencies. All of these modifications generally are based upon a direct multiplication of the TS predicted stress by an empirically developed function usually designated as a_F. Swanson[3] has proposed a theory based upon a strain-softening concept and Martin[2] has proposed a theory based upon a modified shift function which depends not only on temperature but on stress magnitude. All of the theories are in an early stage of development and have shown some success under limited test conditions.

In this Note, a thermoviscoelastic constitutive theory is proposed in which the shift factor a_T in the TS theory is assumed to depend not only upon the instantaneous temperature T but also upon the instantaneous rate of temperature change $\dot{T}$.

Material Characterization

Transient thermal tests were conducted on a typical PBAA composite solid propellant material with approximately 85% solids loading. The material stiffness was characterized in a standard manner using stress relaxation tests. The reference time modulus was modeled mathematically using a series of exponentials of the form

$$E(t) = E_\infty + \sum_{i=1}^{15} E_i \exp(-t/\tau_i) \tag{1}$$

Presented as Paper 82-1098 at the AIAA/SAE/ASME 18th Joint Propulsion Conference, Cleveland, Ohio, June 21-23, 1982; submitted Aug. 13, 1982; revision received Jan. 3, 1983.

*Professor and Head, Department of Aerospace Engineering. Member AIAA.

where E_∞ is the equilibrium modulus estimated to be 150 psi and the constants E_i and τ_i are contained in Table 1.

The shift factor a_T was determined experimentally by shifting the stress relaxation data to form a smooth curve at the reference temperature of 75°F. The shift function data was represented mathematically by the expression

$$\log_{10} a_T = \frac{-31.43(T - 75°\text{F})}{904.55 + (T - 75°\text{F})} \tag{2}$$

where T is the local temperature in the grain in degrees Fahrenheit.

Rate-Dependent Shift Function Theory

Thermorheologically simple theory assumes that the molecular processes that govern the constitutive behavior of polymeric materials are governed by a shift factor $a_T(T)$, where T is the instantaneous temperature, independent of temperature history. This assumption may be expressed in terms of the reduced time $\xi(t)$ as

$$\frac{d\xi}{dt} = \frac{1}{a_T[T(t)]} \tag{3}$$

which leads to the familiar relation[4]

$$\xi(t) = \int_0^t \frac{dt'}{a_T[T(t')]} \tag{4}$$

It is proposed that the molecular processes affecting the constitutive behavior depend not only upon the current temperature but upon the rate of temperature change as well. One possible form for the relationship between the reduced time and real time is an additive expression

$$\frac{d\xi}{dt} = f_1(T) + f_2(\dot{T}, T) \tag{5}$$

where a dot over a function denotes differentiation with respect to time. If f_1 and f_2 in Eq. (5) are assumed to have the forms

$$f_1(T) = \frac{1}{a_T[T]}, \qquad f_2(T,\dot{T}) = \dot{T} f_3(T) \tag{6}$$

the proposed modification reduces to the classical TS theory when the temperature is constant.

To determine $f_1(t)$, conventional constant temperature stress relaxation test results may be shifted to obtain $a_T[T]$ for various temperatures. After $a_T[T]$ has been determined by testing under isothermal conditions, $f_2(T,\dot{T})$ may be determined by conducting a series of stress relaxation tests under conditions in which the temperature is changing at a

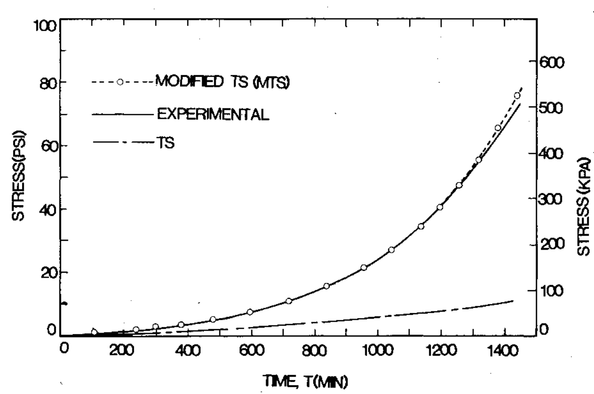

Fig. 1 Transient thermal relaxation stress history for PBAA propellant.

Table 1 Relaxation modulus series constants

i	E_i, psi	τ_i, min
1	18394.5470	3.16228×10^{-9}
2	7207.3604	3.16228×10^{-8}
3	5496.2151	3.16228×10^{-7}
4	2967.1509	3.16228×10^{-6}
5	2387.2908	3.16228×10^{-5}
6	1196.4850	3.16228×10^{-4}
7	1064.0989	3.16228×10^{-3}
8	224.1984	3.16228×10^{-2}
9	402.1704	3.16228×10^{-1}
10	230.1458	3.16228×10^{0}
11	62.5829	$3.16228\times10^{+1}$
12	152.3052	$3.16228\times10^{+2}$
13	32.3993	$3.16228\times10^{+3}$
14	23.2709	$3.16228\times10^{+4}$
15	46.6670	$3.16228\times10^{+5}$

Table 2 Constant straining and cooling test conditions

Test	Strain rate, %/h	Cooling rate, °F/h
1	0	−5.0
2	0.429	−5.0
3	0.429	−4.0
4	0.429	−2.5
5	0.429	−0.5
6	4.290	−2.5
7	1.070	−12.0

constant rate. As noted earlier, the TS theory cannot be used to accurately predict the stresses under these conditions. Consequently, a correction must be made to the reduced time to predict the experimentally measured stresses. The amount of correction to be made to the reduced time $\xi(t)$ can be used to determine $f_2(T,\dot{T})$.

The above procedure was applied to the PBAA propellant by conducting a uniaxial stress relaxation test with $\dot{\epsilon}=0$ and $\dot{T}=-5$°F/h. The measured stress and the stress predicted by TS theory are illustrated in Fig. 1 for the time history of the test.

Using a curve-fitting procedure, the function $f_2(T,\dot{T})$ was determined by calculating the changes to be made in $\xi(t)$ such that the theoretical and experimental stresses agree. The following function for $f_2(T,\dot{T})$ was found to producea theoretically predicted stress history which agreed within graphical plotting accuracy with the experimentally measured stress history,

$$f_3(T,\dot{T})=100a_T[T]^{1.65} \tag{7}$$

Thus the reduced time now has the form

$$\xi(t)=\int_o^t \frac{dt'}{a_T[T(t')]}+\int_o^t \frac{\dot{T}dt'}{100a_T[T(t')]^{1.65}} \tag{8}$$

Analysis of Transient Thermal Tests

To determine the adequacy of the proposed modifications in the TS theory, seven transient thermal tests were conducted on uniaxial specimens as described in Table 2. The tests have been described by Martin.[2] All tests began with zero strain and at a temperature of 75°F. Test 1 is the characterization test described in the previous section. The particular straining and cooling rates used in the tests were selected because of their similarity to the rates encountered in a series of cyclic tests conducted to simulate the thermal conditions in tactical solid rocket motors under thermal storage conditions.[5]

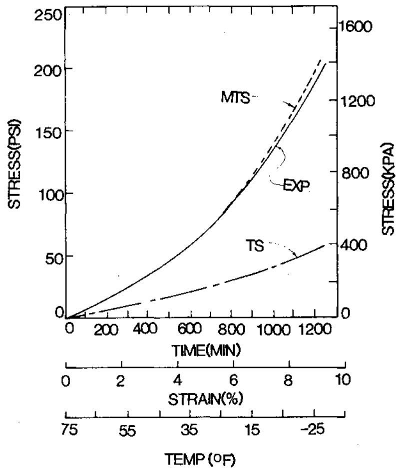

Fig. 2 Similitude test 3 stress history.

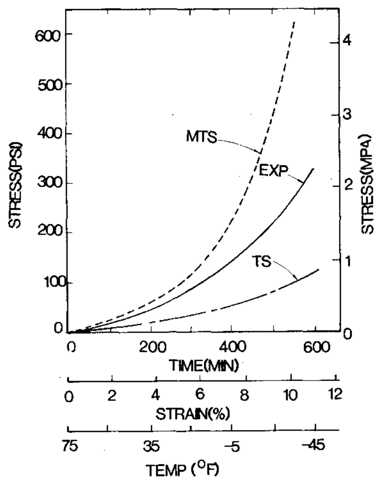

Fig. 3 Similitude test 7 stress history.

Stress predictions for tests 2-5 were excellent. Typical results are shown in Fig. 2, which contains a comparison of the predictions using the theory proposed and the experimental results for test 3. Predictions for tests 6 and 7 were not as good, as indicated, typically, by the results for test 7 in Fig. 3. Tests 6 and 7 were conducted at much higher straining rates and cooling rates, respectively, than the characterization test.

Discussion

The inadequacy of the thermorheologically simple constitutive theory for solid propellant materials under transient thermal conditions is further confirmed by the tests reported here. The modified TS theory proposed here, based upon a correction to the shift function for conditions when $\dot{T}$ is not zero, is only approximately correct. Agreement between the theory and test is good only when the theory is applied to strain rates and cooling rates near where the correction to the shift function was determined. Test results indicate that the characterization process should include stress relaxation tests under a variety of cooling and heating rates to further define the nature of the correction to the shift function.

References

[1]Hufferd, W.L., Lindsey, G., Parmeter, R., and Schapery, R.A., "Predictive Techniques for Failure Mechanisms in Solid Rocket Motors," U.S. Air Force Rocket Propulsion Laboratory, Edwards Air Force Base, Calif., AFRPL TR-79-87, Jan. 1980.

[2]Martin, D.L. Jr., "An Intrinsic Time Shift Function for Solid Propellants," U.S. Army Missile Command, Redstone Arsenal, Ala., RK-81-G, June 1981.